WNE Linear Algebra
Final Exam
Series A

6 February 2021

Problems

Please use separate files for different problems. Please provide the following data in each pdf file

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and the series.**

Each problem is worth 10 marks.

Problem 1.
Let $V = \text{lin}((2, 1, 9, 2), (1, 1, 5, 2), (1, 3, 7, 6))$ be a subspace of $\mathbb{R}^4$.

a) find a basis $\mathcal{A}$ of the subspace V and the dimension of V,
b) for which $t \in \mathbb{R}$ does the vector $v = (1, -2, 2, t) \in \mathbb{R}^4$ belong to V? for each such $t \in \mathbb{R}$ find coordinates of v relative to the basis $\mathcal{A}$.

Problem 2.
Let $V \subset \mathbb{R}^4$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} x_1 + 2x_2 + 5x_3 + 14x_4 = 0 \\ 2x_1 + 3x_2 + 9x_3 + 23x_4 = 0 \end{cases}$$

a) find a basis $\mathcal{A}$ of the subspace V and the dimension of V,
b) complete basis $\mathcal{A}$ to a basis $\mathcal{B}$ of the subspace $W \subset \mathbb{R}^4$, where

$$W = \left\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 + 2x_2 + 5x_3 + 14x_4 = 0\right\}.$$

Problem 3.
Let $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear endomorphism given by the formula

$$\varphi((x_1, x_2, x_3)) = (5x_1 - 4x_2 + 4x_3, 6x_1 - 5x_2 + 6x_3, x_3).$$

a) find the eigenvalues of φ and bases of the corresponding eigenspaces,
b) compute A^{50}, where $A = M(\varphi)_{st}^{st}$.

Problem 4.
Let $\mathcal{A} = ((1, 1, 2), (1, 0, 1), (0, 0, 1))$ be an ordered basis of $\mathbb{R}^3$. Let $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by the matrix

$$M(\varphi)_{\mathcal{A}}^{st} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}.$$

a) find the formula of φ,
b) find the matrix $M(\varphi)_{st}^{\mathcal{A}}$.

Problem 5.
The affine subspaces $E,\ H \subset \mathbb{R}^3$ are given by

$$E = (1,0,2) + \operatorname{lin}((1,0,1)),$$
$$H = (1,1,3) + \operatorname{lin}((0,1,1)).$$

i) describe the affine space E by a system of linear equations,
ii) check if the intersection of E and H is non–empty.

Problem 6.
Consider the following linear programming problem $-2x_3 + 2x_4 + 3x_5 \to \min$ in the standard form with constraints

$$\begin{cases} 2x_1 + x_2 + 12x_3 + 9x_4 + 3x_5 = 16 \\ x_1 + x_2 + 8x_3 + 6x_4 + 4x_5 = 10 \end{cases} \text{and } x_i \geqslant 0 \text{ for } i = 1, \ldots, 5$$

a) which of the sets $\mathcal{B}_1 = \{1,2\}$, $\mathcal{B}_2 = \{3,4\}$, $\mathcal{B}_3 = \{2,5\}$ are basic? Which basic set is basic feasible? Write the corresponding feasible solution.
b) solve the linear programming problem using simplex method.